# Transient Response of Lossy Multiconductor Transmission Line, Terminated in Complex Nonlinear Loads and Illuminated by an External Electromagnetic Field 

Ilham Zerrouk, Hassane Kabbaj, Amine Amharech


#### Abstract

This paper investigates the electromagnetic field coupling to a lossy and planar uniform multiconductor transmission line (MTL), connected with multiple complex nonlinear components such as the Metal Semiconductor Field-Effect Transistor (MESFET) or the Metal Oxide Semiconductor Field-Effect Transistor (MOSFET). Under the assumption of a quasi-transverse electromagnetic (quasi-TEM) field structure, we obtain the expressions of the currents and the voltages induced in the line, using the finite-difference time-domain (FDTD) algorithm. The calculated results using the proposed method show good agreement with results obtained by commercial software PSpice.


Index Terms- Electromagnetic compatibility, FDTD, Incident field, MESFET, Multiconductor Transmission Lines, MOSFET, PSpice

## 1 Introduction

T'HE electromagnetic compatibility (EMC) applied in the Radio Frequency (RF) and microwave components industry is a vital domain which gains a great interest recently. Several phenomena can negatively impact the sophistical circuits among which we can state electromagnetic field coupling, crosstalk and skin effect... In this regard; there is a great concern in the incident electromagnetic field coupling to a circuit board. This phenomenon can seriously deteriorate the signal integrity of the design and cause false switching of waveforms. Furthermore, it can generate spurious signals and potentially damaging the electronic components.

Incorporating the effects of incident electromagnetic fields into interconnect simulation has been a problem of interest for many years. Under the quasi-TEM assumption, these effects are then represented by the distributed sources along transmission lines. Therefore, various techniques have been proposed for solving the transmission-line model with distributed sources. In [1], [2], [3], [4], [5], [6], [7], [8], the authors studied the circuits with complexity of transmission line only or the complexity of the load with frequency or time methods.

In our work we associate the complexity of both line of transmission and load. The idea behind is to illustrate the transient response of a microwave complex circuit, that is composed of the lossy micro-strip MTL connected by the mul-ti-transistors (MESFETs) modeled by their large-signal equivalent schemes [9], [10] and illuminated by an external electromagnetic field.

In this analysis, we use the finite-difference time-domain method [11], [12], which is a direct solution to the Telegrapher's equations in the time-domain and can be applied at complicated problems associated with increasingly higher frequencies. It is proven to be the simplest in solving problems involving electromagnetic wave interactions. In the FDTD approach, both space and time are divided into discrete seg-
ments. This technique was proposed firstly by Yee [13] and then developed by Umashankar and Taflove [14], where responses are supposed to be stable if the increment of time and space are carefully chosen.

The rest of the paper is organized as follows. Section 2 contains an analysis of the Telegrapher's MTL equation illuminated by the external field, where we use the proposed FDTD approach to characterize the line's transient response. In addition, the effect of electromagnetic coupling to multi-MESFETs loads is the principal interest in this paper. The method that is employed for the analysis of the nonlinear behavior is the Newton-Raphson. Numerical results and discussions are presented in section 3, followed by conclusions in section 4.

## 2 ANALYSIS OF MTL ILLUMINATED BY AN EXTERNAL FIELD

Micro strip line is taken into account in this paper because it's one of the popular and simplest types of planar structures that are appropriate for many applications in the microwave industry. Fig. 1 shows such a line illuminated by an incident electromagnetic field, represented by a uniform plane wave from far-field source [1]:


(b)

Fig. 1. (a) Incident and azimuth angles, (b) Polarization angle.

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As can be seen in Fig. 2, the effect of the incident field into the transmission line is represented by equivalent distributed voltage and current generators located along the line [1]:


Fig. 1. Transmission line $\pi$ spatial cell with illumination.

Consider a $(\mathrm{n}+1)$ conductor, uniform transmission line; the Telegrapher's equations of the MTL in the terms of total voltages, illuminated by an incident field are [1]:

$$
\begin{align*}
& \frac{d \boldsymbol{V}(z, t)}{d z}+\boldsymbol{L}(z) \frac{d \boldsymbol{I}(z, t)}{d t}+\boldsymbol{R} \boldsymbol{I}(z, t)=\boldsymbol{V}_{F}(z, t)  \tag{1}\\
& \frac{d \boldsymbol{I}(z, t)}{d z}+\boldsymbol{C}(z) \frac{d \boldsymbol{V}(z, t)}{d t}+\boldsymbol{G} \boldsymbol{V}(z, t)=\boldsymbol{I}_{F}(z, t) \tag{2}
\end{align*}
$$

Where

$$
\begin{equation*}
\boldsymbol{V}_{F}(z, t)=\boldsymbol{E}_{L}(z, t)-\frac{d \boldsymbol{E}_{T}(z, t)}{d z} \tag{3}
\end{equation*}
$$

$\boldsymbol{I}_{F}(z, t)=-\boldsymbol{C} \frac{d \boldsymbol{E}_{T}(z, t)}{d t}$


The ( $\mathrm{n} \times \mathrm{n}$ ) matrices $\mathbf{R}, \mathbf{L}, \mathbf{G}$ and $\mathbf{C}$ are the resistance, inductance, conductance and capacitance per unit length characterizing the MTL. Moreover, V and I are ( $\mathrm{n} \times 1$ ) vectors of the total voltages (with respect to the reference conductor) and currents matrices along the MTL.

The $(\mathrm{n} \times 1)$ vectors, $\mathrm{V}_{\mathrm{F}}(\mathrm{z}, \mathrm{t})$ and $\mathrm{I}_{\mathrm{F}}(\mathrm{z}, \mathrm{t})$, are distributed voltage and current sources vectors, due to the incident field [1].

The terms $E_{T}(z, t)$ and $E_{L}(z, t)$ are $(n \times 1)$ vectors having the components of the incident electric field that are transverse to the line and parallel to the line conductors, respectively, with the line conductors removed [1]. The term $\mathrm{E}_{\mathrm{T}}(\mathrm{z}, \mathrm{t})$ is given by:

- Ilham Zerrouk is currently aPhD student in microwave electronic at Faculty of science and Technology of Fez in Morocco. E-mail: ilham.zerouk@usmba.ac.ma
- Hassane Kabbaj is currently a research professor at the Faculty of science and Technology of Fez in Morocco, Its focus is the electromagnetic compatibility applied to Transmission Lines. E-mail: hkabbaj2@yahoo.fr
- Amine Amharech is currently a research professor at the National School of Applied Sciences of Khouribga in Morocco, Its focus is the electromagnetic compatibility applied to Transmission Lines. E-mail: amineetudes@yahoo.fr
$\boldsymbol{E}_{T}(z, t)=\int_{o}^{h} E_{x}^{i, t o t a l} d x$
It's expressed by the integral of the component of the incident electric field intensity vector that is in a plane transverse to the line conductors.

The quantity $\mathrm{E}_{\mathrm{L}}(\mathrm{z}, \mathrm{t})$, is the difference between the longitudinal components of the incident electric field intensity vector along the position of the $\mathrm{i}^{\text {th }}$ conductor and along the position of the reference conductor. This quantity is written as:
$\boldsymbol{E}_{L}(z, t)=E_{z}^{i, \text { total }}(h, z, t)-E_{z}^{i, \text { total }}(o, z, t)$

### 2.1 Solving Transmission Line Equation by FDTD Method

The finite-difference time-domain (FDTD) method is a directly way of obtaining the time domain response of MTL equations; it has been used successfully to analyze the MTL equations in the case of the line is considered lossy which is the case for us in this paper.

The MTL equations are discretized in position along the line and in time using the leap-frog scheme shown in Fig. 3:


Fig. 1(a) Illustration of FDTD discretization, and (b) the locations of the solution variables on the line.

In order, to illustrate the method, we divide the line into ndz segments each of equal length $\Delta z$ as shown in Fig. 3b. Similarly, the total solution time is divided into ndt sections of equal length $\Delta \mathrm{t}$. To provide the accuracy of the discretization, the voltage points $\mathrm{V}_{1}, \mathrm{~V}_{2}, \mathrm{~V}_{3}, \ldots, \mathrm{~V}_{\text {ndz }+1}$ and the current points $\mathrm{I}_{1}$, $\mathrm{I}_{2}, \ldots, \mathrm{I}_{\mathrm{ndz}}$ are interlaced as shown in Figure 3b. Each voltage and adjacent current solution point are separated by $\Delta z / 2$. In addition, the time points are also interlaced, and each voltage time point and adjacent current time point are separated by $\Delta t / 2$ [15], [16], [17]. The finite difference approximations to the difference equations (1) and (2) become:

$$
\begin{align*}
& \frac{\boldsymbol{V}_{k+1}^{n+1}-\boldsymbol{V}_{k}^{n+1}}{\Delta \mathrm{z}}=-\boldsymbol{L} \frac{\boldsymbol{I}_{k}^{n+\frac{3}{2}}-\boldsymbol{I}_{k}^{n+\frac{1}{2}}}{\Delta \mathrm{t}}-\boldsymbol{R} \frac{\boldsymbol{I}_{k}^{n+\frac{3}{2}}+\boldsymbol{I}_{k}^{n+\frac{1}{2}}}{2} \\
& -\frac{\boldsymbol{E}_{T, k+1}^{n+1}-\boldsymbol{E}_{T, k}^{n+1}}{\Delta \mathrm{z}}+\frac{\boldsymbol{E}_{L, k}^{n+\frac{3}{2}}+\boldsymbol{E}_{L, k}^{n+\frac{1}{2}}}{2}  \tag{7}\\
& \frac{\boldsymbol{I}_{k}^{n+\frac{1}{2}}-\boldsymbol{I}_{k-1}^{n+\frac{1}{2}}}{\Delta \mathrm{z}}=-\boldsymbol{C}\left(\frac{\boldsymbol{V}_{k}^{n+1}-\boldsymbol{V}_{k}^{n}}{\Delta \mathrm{t}}+\frac{\boldsymbol{E}_{T, k}^{n+1}-\boldsymbol{E}_{T, k}^{n}}{\Delta \mathrm{t}}\right)  \tag{8}\\
& -\boldsymbol{G} \frac{\boldsymbol{V}_{k}^{n+1}+\boldsymbol{V}_{k}^{n}}{2}
\end{align*}
$$

Rearranging these gives:
$\boldsymbol{V}_{k}^{n+1}=\left(\frac{\boldsymbol{C}}{\Delta t}+\frac{\boldsymbol{G}}{2}\right)^{-1}\left[\left(\frac{\boldsymbol{C}}{\Delta t}-\frac{\boldsymbol{G}}{2}\right) \boldsymbol{V}_{k}^{n}-\frac{\boldsymbol{C}}{\Delta t}\left(\boldsymbol{E}_{T, k}^{n+1}-\boldsymbol{E}_{T, k}^{n}\right)\right.$
$\left.-\frac{1}{\Delta z}\left(\boldsymbol{I}_{k}^{n+\frac{1}{2}}-\boldsymbol{I}_{k-1}^{n+\frac{1}{2}}\right)\right]$
For $\mathrm{k}=2,3 \ldots . \mathrm{ndz}$
$\boldsymbol{I}_{k}^{n+\frac{3}{2}}=\left(\frac{\boldsymbol{L}}{\Delta t}+\frac{\boldsymbol{R}}{2}\right)^{-1}\left[\left(\frac{\boldsymbol{L}}{\Delta t}-\frac{\boldsymbol{R}}{2}\right) \boldsymbol{I}_{k}^{n+\frac{1}{2}}+\frac{1}{2}\left(\boldsymbol{E}_{L, k}^{n+\frac{3}{2}}+\boldsymbol{E}_{L, k}^{n+\frac{1}{2}}\right)\right.$
$\left.-\frac{1}{\Delta \mathrm{z}}\left(\left(\boldsymbol{V}_{k+1}^{n+1}-\boldsymbol{V}_{k}^{n+1}\right)+\left(\boldsymbol{E}_{T, k+1}^{n+1}-\boldsymbol{E}_{T, k}^{n+1}\right)\right)\right]$

For $\mathrm{k}=1,2, \ldots . \mathrm{ndz}$
Where we denote
$\boldsymbol{V}_{k}^{n} \equiv \boldsymbol{V}[(k-1) \Delta \mathrm{z}, \mathrm{n} \Delta \mathrm{t}]$
$\boldsymbol{I}_{k}^{n} \equiv \boldsymbol{I}\left[\left(k-\frac{1}{2}\right) \Delta \mathrm{z}, \mathrm{n} \Delta \mathrm{t}\right]$
The transverse incident field sources, $\mathrm{E}_{\mathrm{T}}(\mathrm{z}, \mathrm{t})$, are to be considered at the voltage positions, whereas the longitudinal field sources, $E_{L}(z, t)$, are to be evaluated at the current positions, i.e.

$$
\begin{align*}
& \boldsymbol{E}_{T, k}^{n} \equiv \boldsymbol{E}_{T}[(k-1) \Delta \mathrm{z}, \mathrm{n} \Delta \mathrm{t}] \\
& \boldsymbol{E}_{L, k}^{n} \equiv \boldsymbol{E}_{L}\left[\left(k-\frac{1}{2}\right) \Delta \mathrm{z}, \mathrm{n} \Delta \mathrm{t}\right] \tag{12}
\end{align*}
$$

Where subscript $k$ denotes the incremental position and superscript n denotes the incremental time.

Next, consider the incorporation of the terminal conditions, equations (1) and (2) are solved to give the recursion relations at each end of the line, in the case of the linear termination.

Hence, the MTL equation is discretized at the source as:
$\boldsymbol{V}_{1}^{n+1}=A^{-1}\left[B \boldsymbol{V}_{1}^{n}-\frac{\boldsymbol{C}}{\Delta \mathrm{t}}\left(\boldsymbol{E}_{T, 1}^{n+1}-\boldsymbol{E}_{T, 1}^{n}\right)\right.$
$\left.+\frac{1}{\Delta \mathrm{z}}\left(\boldsymbol{R}_{s}^{-1}\left(\boldsymbol{V}_{s}^{n+1}+\boldsymbol{V}_{s}^{n}\right)-2 \boldsymbol{I}_{1}^{n+\frac{1}{2}}\right)\right]$

Where $\quad \boldsymbol{A}=\frac{\boldsymbol{C}}{\Delta \mathrm{t}}+\frac{\boldsymbol{G}}{2}-\frac{1}{\Delta \mathrm{z}} \boldsymbol{R}_{s}^{-1} \quad, \boldsymbol{B}=\frac{\boldsymbol{C}}{\Delta \mathrm{t}}-\frac{\boldsymbol{G}}{2}-\frac{1}{\Delta \mathrm{z}} \boldsymbol{R}_{s}^{-1}$
Similarly, MTL equation is discretized at the load as:

$$
\begin{align*}
& \boldsymbol{V}_{n d z+1}^{n+1}=\left(\frac{\boldsymbol{C}}{\Delta t}+\frac{\boldsymbol{G}}{2}\right)^{-1}\left[\left(\frac{\boldsymbol{C}}{\Delta t}-\frac{\boldsymbol{G}}{2}\right) \boldsymbol{V}_{n d z+1}^{n}-\frac{\boldsymbol{C}}{\Delta \mathrm{t}}\left(\boldsymbol{E}_{T, n d z+1}^{n+1}-\boldsymbol{E}_{T, n d z+1}^{n}\right)\right. \\
& \left.+\frac{2}{\Delta \mathrm{z}}\left(\frac{\boldsymbol{I}_{L}^{n+1}+\boldsymbol{I}_{L}^{n}}{2}-\boldsymbol{I}_{n d z}^{n+\frac{1}{2}}\right)\right] \tag{15}
\end{align*}
$$

$\mathrm{I}_{\mathrm{L}}$ denotes the current load, it can be written as:
$\boldsymbol{I}_{L}^{n+1}=\boldsymbol{V}_{n d z+1}^{n+1} \cdot \boldsymbol{R}_{L}^{-1} \quad$ and $\quad \boldsymbol{I}_{L}^{n}=\boldsymbol{V}_{n d z+1}^{n} \cdot \boldsymbol{R}_{L}^{-1}$
Replacing these in the equation (15) gives:
$\boldsymbol{V}_{n d z+1}^{n+1}=\boldsymbol{D}^{-1}\left[\boldsymbol{E} \boldsymbol{V}_{n d z+1}^{n}+\frac{2}{\Delta \mathbf{z}} \boldsymbol{I}_{n d z}^{n+\frac{1}{2}}\right.$
$\left.-\frac{\boldsymbol{C}}{\Delta \mathrm{t}}\left(\boldsymbol{E}_{T, n d z+1}^{n+1}-\boldsymbol{E}_{T, n d z+1}^{n}\right)\right]$
Where $\quad \boldsymbol{D}=\frac{\boldsymbol{C}}{\Delta \mathrm{t}}+\frac{\boldsymbol{G}}{2}+\frac{1}{\Delta \mathrm{z}} \boldsymbol{R}_{L}^{-1} \quad, \boldsymbol{E}=\frac{\boldsymbol{C}}{\Delta \mathrm{t}}-\frac{\boldsymbol{G}}{2}-\frac{1}{\Delta \mathrm{z}} \boldsymbol{R}_{L}^{-1}$
Once again, for stability, the position and time discretization must satisfy the current stability condition which expressed as [18]:
$\Delta t \leq \frac{\Delta z}{v_{p \max }}$
Where $v_{\text {pmax }}$ is the maximal propagation velocity in the MTL.

### 2.2 Complex Nonlinear Load

In this paper, we are interested in the MTL connected by nonlinear loads such as the multi-MESFETs in common-source

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configuration. We are focused in the nonlinear comportment of the MESFET. In the figure below, we adopt his intrinsic large signal taken from [9], [10]:


Fig. 4. Intrinsic large-signal model of MESFET
Where the principal nonlinearities in this model are represented by [19], [20], [21], [22], [23], [24]:

$$
\begin{align*}
& \boldsymbol{C}_{g s}\left(\boldsymbol{V}_{g}(\mathrm{t})\right)=\boldsymbol{C}_{g s 0}\left(\sqrt{1-\left(\boldsymbol{V}_{b}^{-1} \boldsymbol{V}_{g}(\mathrm{t})\right)}\right)^{-1} \text { For } \boldsymbol{V}_{g}<\boldsymbol{V}_{b}  \tag{20}\\
& \boldsymbol{C}_{g s}\left(\boldsymbol{V}_{g}(\mathrm{t})\right)=\frac{\boldsymbol{C}_{g s 0}}{\left(1-F_{C}\right)^{\frac{3}{2}}}\left(1-\frac{3}{2} F_{C}+\frac{1}{2} \boldsymbol{V}_{b}^{-1} \boldsymbol{V}_{g}(\mathrm{t})\right) \tag{21}
\end{align*}
$$

And
$\left.\boldsymbol{I}_{d s}\left(V_{g}(t)\right)_{W_{d s}}(t)\right)=\left(A_{0}+{ }_{1 g}(t)+\right.$
$+\boldsymbol{A}_{2} \boldsymbol{V}_{g}^{2}\left(\boldsymbol{d}_{\mathrm{d}} \mathbf{I} \mathbf{4} \boldsymbol{A}_{3} \boldsymbol{V}_{g}^{3}(t)\right) \tanh (\quad \mathrm{ds}(t))$
Where Vb is the built-in potential of the Schottky gate and $\delta$ is a $(\mathrm{n} \times 1)$ vector of 1 .

In this case, the current load $\left(\mathrm{I}_{\mathrm{L}}\right)$ is replaced by its nonlinear expression according to the nonlinear load illustrated in Fig. 4:

$$
\begin{equation*}
\boldsymbol{I}_{L}(\boldsymbol{t})=\boldsymbol{C}_{g s}\left(\boldsymbol{V}_{g}(t)\right) \frac{d \boldsymbol{V}_{g}(t)}{d t}+\boldsymbol{C}_{g d}\left[\frac{d \boldsymbol{V}_{\mathrm{ndz}+1}(t)}{d t}-\frac{d \boldsymbol{V}_{T}(t)}{d t}\right] \tag{23}
\end{equation*}
$$

In terms of finite difference, IL can be expressed as:

$$
\begin{align*}
& \boldsymbol{I}_{L}^{n}=\boldsymbol{C}_{g s}\left(\boldsymbol{V}_{g}^{n}\right)\left[\frac{\boldsymbol{V}_{g}^{n}-\boldsymbol{V}_{g}^{n-1}}{\Delta t}\right]+\boldsymbol{C}_{g d}\left[\frac{\boldsymbol{V}_{n d+1}^{n}-\boldsymbol{V}_{n d z+1}^{n-1}}{\Delta t}-\frac{\boldsymbol{V}_{T}^{n}-\boldsymbol{V}_{T}^{n-1}}{\Delta t}\right]  \tag{24}\\
& \boldsymbol{I}_{L}^{n+1}=\boldsymbol{C}_{g s}\left(\boldsymbol{V}_{g}^{n+1}\right)\left[\frac{\boldsymbol{V}_{g}^{n+1}-\boldsymbol{V}_{g}^{n}}{\Delta t}\right]+\boldsymbol{C}_{g d}\left[\frac{\boldsymbol{V}_{n d+1+1}^{n+1}-\boldsymbol{V}_{n d+1}^{n}}{\Delta t}-\frac{\boldsymbol{V}_{T}^{n+1}-\boldsymbol{V}_{T}^{n}}{\Delta t}\right] \tag{25}
\end{align*}
$$

And $\mathrm{V}_{\mathrm{nd} z+1}$ is the expression of the far end voltage which is given by the equation (15) as mentioned above:

$$
\begin{aligned}
& \boldsymbol{V}_{n d z+1}^{n+1}=\left(\frac{\boldsymbol{C}}{\Delta t}+\frac{\boldsymbol{G}}{2}\right)^{-1}\left[\left(\frac{\boldsymbol{C}}{\Delta t}-\frac{\boldsymbol{G}}{2}\right) \boldsymbol{V}_{n d z+1}^{n}\right. \\
& \left.-\frac{\boldsymbol{C}}{\Delta t}\left(\boldsymbol{E}_{T, n d z+1}^{n+1}-\boldsymbol{E}_{T, n d z+1}^{n}\right)-\frac{2}{\Delta z}\left(\frac{\boldsymbol{I}_{L}^{n+1}+\boldsymbol{I}_{L}^{n}}{2}-\boldsymbol{I}_{n d z}^{n+\frac{1}{2}}\right)\right]
\end{aligned}
$$

Replacing $\mathrm{I}_{\mathrm{L}}$ in $\mathrm{V}_{\mathrm{ndz}+1}$ gives:

$$
\begin{align*}
& \boldsymbol{V}_{n d z+1}^{n+1}=\left(\frac{\boldsymbol{C}}{\Delta t}+\frac{\boldsymbol{G}}{2}\right)^{-1}\left[\left(\frac{\boldsymbol{C}}{\Delta t}-\frac{\boldsymbol{G}}{2}\right) \boldsymbol{V}_{n d z+1}^{n}-\frac{\boldsymbol{C}}{\Delta t}\left(\boldsymbol{E}_{T, n d z+1}^{n+1}-\boldsymbol{E}_{T, n d z+1}^{n}\right)\right. \\
& +\frac{2}{\Delta z} \boldsymbol{I}_{n d z}^{n+\frac{1}{2}}-\frac{1}{\Delta z}\left(\frac{\boldsymbol{C}_{g s}}{\Delta t}\left(\boldsymbol{V}_{g}^{n+1}-\boldsymbol{V}_{g}^{n-1}\right)+\frac{\boldsymbol{C}_{g d}}{\Delta t}\left(\boldsymbol{V}_{n d z+1}^{n+1}-\boldsymbol{V}_{n d z+1}^{n-1}\right)\right.  \tag{26}\\
& \left.\left.-\frac{\boldsymbol{C}_{g d}}{\Delta t}\left(\boldsymbol{V}_{T}^{n+1}-\boldsymbol{V}_{T}^{n-1}\right)\right)\right]
\end{align*}
$$

It's observed in the scheme Fig. 4 that $\mathrm{V}_{\mathrm{ndz}+1}$ can be also written as:
$\boldsymbol{V}_{n d z+1}^{n+1}=\boldsymbol{V}_{g}^{n+1}+\boldsymbol{R}_{i} \boldsymbol{C}_{g s}\left(\boldsymbol{V}_{g}^{n+1}\right)\left[\frac{\boldsymbol{V}_{g}^{n+1}-\boldsymbol{V}_{g}^{n}}{\Delta \mathrm{t}}\right]$
Replacing $\mathrm{V}_{\mathrm{ndz}+1}$ of the equation (27) in (26) gives:
$\boldsymbol{V}_{\boldsymbol{T}}^{n+1}=\boldsymbol{V}_{T}^{n-1}-\boldsymbol{V}_{n d z+1}^{n-1}-\left(\frac{\boldsymbol{C}}{\Delta t}-\frac{\boldsymbol{G}}{2}\right)\left(\frac{\boldsymbol{C}_{g d}}{\Delta t \Delta z}\right)^{-1} \boldsymbol{V}_{n d z+1}^{n}$
$+\left(\boldsymbol{C} \boldsymbol{C}_{g d}^{-1} \Delta z\right)\left(\boldsymbol{E}_{T, n d z+1}^{n+1}-\boldsymbol{E}_{T, n d z+1}^{n}\right)$
$-\left(2 \Delta t \boldsymbol{C}_{g d}^{-1}\right) \boldsymbol{I}_{n d z}^{n+\frac{1}{2}}-\left(\boldsymbol{C}_{g s} \boldsymbol{C}_{g d}^{-1}\right)\left(\boldsymbol{V}_{g}^{n+1}-\boldsymbol{V}_{g}^{n-1}\right)$
$+\left(1+\left(\frac{\boldsymbol{C}}{\Delta t}+\frac{\boldsymbol{G}}{2}\right)\left(\frac{\boldsymbol{C}_{g d}}{\Delta t \Delta z}\right)^{-1}\right)\left(\boldsymbol{V}_{g}^{n+1}+\frac{\boldsymbol{R}_{i} \boldsymbol{C}_{g s}}{\Delta t}\left(\boldsymbol{V}_{g}^{n+1}-\boldsymbol{V}_{g}^{n}\right)\right)$
According to the scheme in Figure 4, $V_{\pi}^{\infty+2}$ can be also expressed by:
$\boldsymbol{V}_{\boldsymbol{T}}^{n+1}=-\boldsymbol{R}_{\mathrm{L}} \boldsymbol{i}^{n+1}=-\boldsymbol{R}_{L}\left(\boldsymbol{I}_{C d s}^{n+1}-\boldsymbol{I}_{C g d}^{n+1}+\boldsymbol{I}_{G d}^{n+1}+\boldsymbol{I}_{d s}^{n+1}\right)$
With $\quad \boldsymbol{I}_{C d s}^{n+1}=\frac{\boldsymbol{C}_{d s}}{\Delta t}\left(\boldsymbol{V}_{T}^{n+1}-\boldsymbol{V}_{T}^{n}\right)$

$$
\boldsymbol{I}_{C g d}^{n+1}=\frac{\boldsymbol{C}_{g d}}{\Delta t}\left(\boldsymbol{V}_{n d z+1}^{n+1}-\boldsymbol{V}_{n d z+1}^{n}-\boldsymbol{V}_{T}^{n+1}+\boldsymbol{V}_{T}^{n}\right)
$$

$$
\boldsymbol{I}_{G d}^{n+1}=\boldsymbol{G}_{\boldsymbol{d}} \boldsymbol{V}_{T}^{n+1}
$$

And

The $E_{T}^{n+2}$ and $E_{T}^{x}$ vectors are computed outside the main algorithm of currents and voltages computation. Besides, all values determined at the anterior time with " $n$ " or " $n-1$ " subscript are known. Subsequently, the only unknown term in all these equations is $V_{s}^{n+2}$, which can be solved by using the New-ton-Raphson method as shown below:
$\boldsymbol{H} \equiv-\boldsymbol{V}_{g}^{n+1}+f\left(\boldsymbol{V}_{g}^{n+1}\right)=0$
Where $f$ is a term of $V_{a}^{s+1}$ and where the nonlinear compo-
nents are collected. The matrix $V_{\underset{\sim}{n+1}}^{n}$ is found by solving the following equation:
$\left(\boldsymbol{V}_{g}^{n+1}\right)^{m+1}=\left(\boldsymbol{V}_{g}^{n+1}\right)^{m}-\left(\boldsymbol{J}_{\boldsymbol{H}}\right)^{-1} \boldsymbol{H}$
Where $\left.\boldsymbol{J}_{\boldsymbol{H}}=\frac{d \boldsymbol{H}}{d \boldsymbol{V}_{g}} \right\rvert\, \boldsymbol{V}_{g}=\left(\boldsymbol{V}_{g}\right)^{m}$

Once $V_{V}^{n+1}$ found, we can determine $V_{T}^{n+1}$ and $V_{m a x+2}^{n+1}$.

## 3 NUMERICAL RESULTS

The numerical solution and the transient response of the multiconductor transmission lines discussed above will be evaluated by MATLAB simulations in this section and we will consider two case studies. The first one is simple nonlinear loads as diodes and the second is complex nonlinear loads as MESFETs. In these cases the line is illuminated by an incident electromagnetic field, whose incident, azimuth and polarization angles are respectively: $\theta_{\mathrm{p}}=90^{\circ}, \varphi_{\mathrm{p}}=-90^{\circ}, \theta_{\mathrm{E}}=90^{\circ}$.

### 6.1 First Example

As the first case study, we choose a micro-strip transmission line consists of 2 lossy conductors plus a reference conductor, where the first and second conductors are connected to the ground at the near end by resistances $\mathrm{R}_{\mathrm{S} 1}=50 \Omega$ and $\mathrm{R}_{\mathrm{S} 2}=75 \Omega$, respectively. This line is chosen to be achieved by the dimensions of $\mathrm{l}=3 \mathrm{~cm}, \mathrm{w}=20 \mu \mathrm{~m}$ and $\mathrm{S}=24,64 \mu \mathrm{~m}$ as shown in Fig. 5. Moreover, it's on a Printed Circuit Board (PCB) with a Silicon substrate $\left(\varepsilon_{\mathrm{r}}=12\right)$ of thickness is $\mathrm{h}=100 \mu \mathrm{~m}$. Thus, the per-unit length parameter matrices are computed using the numerical method of [1]:
$\boldsymbol{L}=\left(\begin{array}{lll}0.735474 & 0.310351 \\ 0.310351 & 0.735474\end{array}\right) n H / m$
$\boldsymbol{C}=\left(\begin{array}{cr}30.2524 & -11.7363 \\ -11.7363 & 30.2525\end{array}\right) p F / m$
We consider the line to have zero conductance $(\mathbf{G}=0)$ and the per-unit-length resistance of each conductor is computed as:
$\boldsymbol{R}=\left(\begin{array}{c}172.4310 \\ 0 \\ 172.431\end{array}\right) \Omega / m$
The incident electromagnetic field is illustrated by a uniform incident plane wave with rise time $\tau_{r}=1 \mathrm{~ns}$ and amplitude $E_{0}=10 \mathrm{KV} / \mathrm{m}$.
We consider a nonlinear termination at the far end; this is obtained by inserting a diode in series with the $10 \Omega$ load at the right end of the line as shown in Fig. 5. The diode is characterized by:

$$
i_{D}=I_{S}\left(\exp \left(\frac{v_{D}}{V_{T}}\right)-1\right)
$$

Where $\mathrm{I}_{\mathrm{S}}=10 \mathrm{nA}$ and $\mathrm{V}_{\mathrm{T}}=25 \mathrm{mV}$.


Fig. 6. Input and output voltages in the case of the line loaded by diode. (a) FDTD and (b) PSpice.

### 6.1 Second Example

In this example, we study the coupling electromagnetic effect in a circuit composed of five lossy conductors plus the reference conductor of 2.54 cm in the length, loaded by five identical transistors configured in common-source and illuminated by an external source ramp of 10 KV in amplitude, with a rise time varying between of 100 ps and 1 ns . We consider the resistances of the near ends (NE) as well as of the far ends (FE) equal to $50 \Omega$ as presented in Fig. 7.

The parameters of the transistors are:
$\mathbf{A}_{0}=0.001043$. Id, $\quad \mathbf{A}_{1}=0.0011745$. Id, $\quad \mathbf{A}_{2}=-0.0001274$. Id,
$\mathbf{A}_{3}=-0.001 . \operatorname{Id}, \mathbf{a}=1.6 . \mathbf{I d}, \mathbf{F}_{\mathrm{C}}=0.5$

$$
\begin{aligned}
& \mathbf{G}_{\mathrm{d}}=0.2 . \operatorname{Id}(\mathrm{mS}), \mathrm{R}_{\mathrm{i}}=1 . \operatorname{Id}(\Omega), \mathbf{V}_{\mathrm{b}}=0.7 . \operatorname{Id}(\mathrm{V}), \mathbf{C}_{\mathrm{gs} 0}=2 . \operatorname{Id}(\mathrm{pF}), \\
& \mathbf{C}_{\mathrm{ds}}=0.6 . \operatorname{Id}(\mathrm{pF}), \mathbf{C}_{\mathrm{gd}}=1.5 . \operatorname{Id}(\mathrm{pF}) .
\end{aligned}
$$

Where Id is the identity matrix:

$$
\mathbf{I d}=\left(\begin{array}{c}
10000 \\
01000 \\
00100 \\
00010 \\
00001
\end{array}\right)
$$



Fig. 7. Configuration of the MTL loaded by multi-MESFETs.

The line per-unit length parameters taken from [1] are:

$$
\boldsymbol{L}=\left(\begin{array}{ccccc}
0.734067 & 0.307438 & 0.184249 & 0.121470 & 0.0849788 \\
0.307438 & 0.729398 & 0.305467 & 0.183377 & 0.121470 \\
0.184249 & 0.305467 & 0.728728 & 0.305467 & 0.184249 \\
0.121470 & 0.183377 & 0.305467 & 0.729399 & 0.307438 \\
0.084978 & 0.121470 & 0.184249 & 0.307438 & 0.734067
\end{array}\right) n H / m
$$

$$
C=\left(\begin{array}{ccccc}
30.4509 & -10.8466 & -1.95855 & -0.680753 & -0.400583 \\
-10.8466 & 34.4761 & -10.1477 & -1.74112 & -0.680753 \\
-1.95855 & -10.1477 & 34.5876 & -10.1477 & -1.95855 \\
-0.680753 & -1.74112 & -10.1477 & 34.4761-10.8466 \\
-0.400583 & -0.680753 & -1.95855 & -10.8466 & 30.4508
\end{array}\right) p F / m
$$

And the effect of series resistance is represented by:

$$
\boldsymbol{R}=\left(\begin{array}{cccccc}
172.431 & 0 & 0 & 0 & 0 \\
0 & 172.431 & 0 & 0 & 0 \\
\mathbf{Q} 0 & 172.431 & 0 & 0 \\
0 & 0 & 0 & 172.4310 \\
0 & 0 & 0 & 0 & 172.431
\end{array}\right) \quad m
$$

We note that our choice to present the signals of the $1^{\text {st }}$ and $4^{\text {th }}$ conductors is arbitrary and we compare our method with results simulated by PSpice Software. The comparison results indicate a good level of agreement like is displayed in Fig. 8 and Fig. 9.


Fig. 8. Solution for the line in figure 7 with a rise time of 100 ps . Result (a) of FDTD method, compared with the result (b) obtained by PSpice.

It is observed that if the rise time is increased, the shape of the pulses becomes smooth. Hence, the rise time is a very important factor to ensure that the signal will be stable as shown below:


Fig. 9. Solution for the line in figure 7 with a rise time of 1 ns . Result (c) of FDTD method, compared with the result (d) obtained by PSpice.

It is important to mention that the waveforms are distorted by nonlinear loads. In the case of the line connected by linear loads like the resistance, the signals have a rectangular form as represented in [5] and [25].

The near ends voltages $\left(\mathrm{V}_{\mathrm{NE} 1}, \mathrm{~V}_{\mathrm{NE5}}\right)$ and $\left(\mathrm{V}_{\mathrm{NE} 2,} \mathrm{~V}_{\mathrm{NE} 4}\right)$ are superposed, and the far ends signals $\left(\mathrm{V}_{\mathrm{FE} 1}, \mathrm{~V}_{\mathrm{FE5}}\right),\left(\mathrm{V}_{\mathrm{FE} 2}, \mathrm{~V}_{\mathrm{FE4}}\right)$ are also superposed due to the symmetry of the line and loads. The same observation for the transistor's outputs $\left(\mathrm{V}_{\mathrm{T} 1}, \mathrm{~V}_{\mathrm{T} 5}\right)$ and $\left(\mathrm{V}_{\mathrm{T} 2}, \mathrm{~V}_{\mathrm{T} 4}\right)$ is considered.

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## 4 Conclusion

In this paper, the FDTD method has been used to determine the time-domain response of a MTL having complex nonlinear loads, and excited by an incident uniform plane wave.
The proposed method is efficient and can be applied to solve several electromagnetic problems in complicated systems. We have examined the effect of incident electromagnetic field into the microwave circuits connected by nonlinear/nonlinear complexes components. The FDTD results have been compared to the results obtained by PSpice in the case of circuit illuminated by the incident field wherein the rise time is 1 ns , and in the case of 100 ps . Identical results have been achieved that requires a good choice of time and space discretization to satisfy the stability condition in the FDTD method. It gives an accurate means to solve the transmission line equations and to save resources during simulation.
Simulation results proved the performance of the proposed technique. We have used a PC with a processor (Intel Pentium CPU B960 @ 2.20 GHz ). The algorithm showed a reasonable computation time and an optimal memory use compared to the 3D one.

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